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**ROBUST AND REAL-TIME CONTROL OF  
MAGNETIC BEARINGS FOR SPACE ENGINES**



# ROBUST AND REAL-TIME CONTROL OF MAGNETIC BEARINGS FOR SPACE ENGINES

by  
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## 1. RESEARCH OBJECTIVES AND POTENTIAL IMPACT ON PROPULSION

The rotating machines used in space engines; e.g., orbital transfer vehicles, space shuttle main engines (SSME), operate at much higher speeds compared to those used in ground based engines or aircraft engines. The SSME oxygen turbopumps operate at about 30,000 rpm; and rotational speeds of space engines can be as high as 100,000 rpm. At these high speeds, conventional bearings; e.g., ball bearings, are not reliable. Consequently the main objective of this research program is to develop a highly reliable magnetic bearing system for space engine applications.

In recent years, a number of researchers have developed magnetic bearing systems. However, their main focus has been the applications to relatively low speed engines. Currently, NASA Lewis Research Center is developing magnetic bearings for SSME turbopumps. The control algorithms which have been used are based on either the proportional-integral-derivative control (PID) approach or the linear quadratic (LQ) state space approach. These approaches lead to an acceptable performance only when the system model is accurately known, which is seldom true in practice. For example, the rotor eccentricity, which is a major source of vibration at high speeds, cannot be predicted accurately. Furthermore, the dynamics of a rotor shaft, which must be treated as a flexible system to model the elastic rotor shaft, is infinite dimensional in theory and the controller can only be developed on the basis of a finite number of modes. Therefore, the development of the control system is further complicated by the possibility of closed loop system instability because of residual or uncontrolled modes, the so called spillover problem. Consequently, novel control algorithms for magnetic bearings are being developed to be robust to inevitable parametric uncertainties, external disturbances, spillover phenomenon and noise. Also, as pointed out earlier, magnetic bearings must exhibit good performance at a speed over 30,000 rpm. This implies that the sampling period available for the design of a digital control system has to be of the order of 0.5 milli-seconds. Therefore, feedback coefficients and other required controller parameters have to be computed off-line so that the on-line computational burden is extremely small.

The development of the robust and real-time control algorithms is based on the sliding mode control theory. In this method, a dynamic system is made to move along a manifold of sliding hyperplanes to the origin of the state space. The number of sliding hyperplanes equals that of actuators. The sliding mode controller has two

parts: linear state feedback and nonlinear terms. The nonlinear terms guarantee that the system would reach the intersection of all sliding hyperplanes and remain on it when bounds on the errors in the system parameters and external disturbances are known. The linear part of the control drives the system to the origin of state space. Another important feature is that the controller parameters can be computed off-line. Consequently, on-line computational burden is small.

## 2. CURRENT STATUS AND RESULTS

### 2.1 CONTROL ALGORITHM FOR RIGID ROTOR

First, the flexibility of the rotor shaft has been ignored and the basic understanding of the bearing control system has been obtained. The equations of motion for the generic model of a rotor shaft (Figure 1) have been obtained using Newton and Euler laws. For small displacements, the linearized equations have the following form:

$$\ddot{X} + P\dot{X} = BU + VS_s + D \quad (1)$$

where  $X \in R^4$  and represents the radial displacements at locations of magnetic bearings. Elements of  $4 \times 1$  control input vector  $V$  correspond to directions of the elements of  $C$ . The matrix  $P$  represents the gyroscopic effects. The  $2 \times 1$  vector  $S_s$  is simply  $[\sin \Omega t \quad \cos \Omega t]^T$  where  $\Omega$  is the angular speed of the shaft. The vector  $D$  contains transient disturbance, and it is assumed that upper bound on each element,  $d_{\max}$ , is known. The uncertainty in the model due to the lack of precise knowledge of the rotor eccentricity is contained in the  $4 \times 2$  matrix  $V$ . It is assumed that

$$|\mathcal{V}_{ij} - \hat{\mathcal{V}}_{ij}| \leq F_{ij} \quad (2)$$

where  $\mathcal{V}_{ij}$  is the estimate of  $V_{ij}$  and  $F_{ij}$  is the maximum error, which is considered to be known.

The sliding mode control law is as follows:

$$(BU)_i = -\lambda_i \dot{x}_i - \hat{\mathcal{V}}_{i1} \sin \Omega t - \hat{\mathcal{V}}_{i2} \cos \Omega t - \bar{K}_i \text{sat}\left(\frac{S_i}{\phi_i}\right) \quad (3)$$

where

$$\begin{aligned} S_i &= \dot{x}_i + \lambda_i x_i \\ \bar{K}_i &= F_{i1} |\sin \Omega t| + F_{i2} |\cos \Omega t| + d_{\max} + \eta_i \\ \phi_i &= \bar{K}_i / \lambda_i \\ \lambda_i &> 0 \end{aligned}$$

Here,  $\eta_i$  is a small positive number. Note that  $S_i=0$  defines the sliding line (Figure 2) and  $\phi_i$  is the time varying thickness of the boundary layer, which has been introduced around the sliding line to eliminate the chattering behavior.

A computer program has been developed to simulate the digital implementation of the control law, eq. (3). A representative steady state response of the controlled rotor is shown in Figure 3. The effects of  $\lambda_i$  and sampling period  $T_s$  on the performance of the control system are being examined.

A novel algorithm has been developed to determine the current in the magnetic bearing so that the control force required by equation (3) will be exactly achieved in spite of the nonlinear relationship among the magnetic force, coil current and the air gap. Using this algorithm, the level of maximum current will be estimated for the parameters of space engines.

## 2.2. FLEXIBLE ROTOR DYNAMIC MODEL

A mathematical model of the flexible rotor system has been formulated. The rotor system is modeled as a fixed-free axisymmetric shaft with an unbalanced disk inertia, and supported by two electro-magnetic bearings, or four independent actuators (Figure 1). The equations of motion and boundary conditions shown below are derived by applying the Hamilton's principle.

The equations of motion are:

$$m_{x,tt} + E I_{x,zzzz} + (m_d x_{,tt} - I_p \Omega y_{,zzt} - I_t x_{,zzt}) \delta(z-d) = F_{13} \delta(z-a) + F_{57} \delta(z-b) + p e \Omega^2 \cos \Omega t \delta(z-d) \quad (4)$$

$$m_{y,tt} + E I_{y,zzzz} + (m_d y_{,tt} + I_p \Omega x_{,zzt} - I_t y_{,zzt}) \delta(z-d) = F_{24} \delta(z-a) + F_{68} \delta(z-b) + p e \Omega^2 \sin \Omega t \delta(z-d) \quad (5)$$

The boundary conditions are:

$$\begin{aligned} x(0,t) = x_{,z}(0,t) = y(0,t) = y_{,z}(0,t) &= 0 \\ x_{,zz}(L,t) = x_{,zzz}(L,t) = y_{,zz}(L,t) = y_{,zzz}(L,t) &= 0 \end{aligned} \quad (6) \quad (7)$$

Here,

$\Omega$ = shaft rotational speed	$E$ = elastic modules	$m_d$ = disk mass
$I$ = shaft second moment of area	$m$ = shaft mass per unit length	$L$ = shaft length
$p$ = disk unbalanced mass	$e$ = disk eccentricity	$\delta$ = Delta function
$I_p$ = disk polar moment of inertia	$I_t$ = disk transverse moment of inertia	
$F_{13}$ = bearing force F1-F3	$F_{24}$ = bearing force F2-F4	
$F_{57}$ = bearing force F5-F7	$F_{68}$ = bearing force F6-F8	
$(\cdot),t$ = partial derivative with respect to time		
$(\cdot),z$ = partial derivative with respect to space coordinate		

Space engine rotor models developed by other researchers will be examined. The important results from these previous analyses will be used to adjust and tune the above model to reflect the space engine rotor dynamic characteristics. The final model will be used as the basis for the dynamic analysis and control algorithm synthesis of flexible rotors.

## 2.3 EXPERIMENTAL DESIGN

In parallel to the analytical efforts, an experiment has been planned and designed to validate the rotor model, implement the control algorithm, and verify the theoretical predictions. A rotor fixture from Bently Nevada Company (Figure 4) has been specified and purchased. The fixture consists of a shaft with lumped disk inertia. The unbalanced force of the rotor can be adjusted through insertion of weights into the disks. The shaft speed is controlled by an AC motor. Measurements of the transverse vibration of the rotor are performed by using non-contact displacement probes. Two magnetic bearings are being designed and fabricated by Magnetic Bearing Corporation. The rotor fixture is presently being modified to incorporate the magnetic bearings. A PC-based micro-processor control system has been specified and is being set up as the main controller for the control algorithm implementation.

### 3. PROPOSED WORK FOR COMING YEAR

#### 3.1 CONTROL ALGORITHM FOR FLEXIBLE ROTOR

The number of vibratory modes of a flexible rotor is infinite in theory and extremely large in practice. Since it is not practical to control vibration in all the modes, the controller will be designed on the basis of a finite number of modes, which is termed as controlled modes. The remaining higher frequency modes are called 'residual modes'. Since magnetic bearings will provide control forces in four independent directions, the effective numbers of actuators will be less than the number of controlled modes,  $N$ . In this case, sliding hyperplanes will have to be taken as  $S = EX$  where  $S$  and  $X$  are respectively  $4 \times 1$  and  $2N \times 1$  vectors, and  $E$  is a full matrix unlike the situation for the rigid rotor. Furthermore, all the elements of the state vector  $X$  cannot be directly measured. Consequently, these states have to be obtained using an observer. The controller design will essentially involve an appropriate choice of the matrix  $E$  and the observer gain matrix. Using the singular value robustness tests, these matrices will be chosen such that the closed loop system is asymptotically stable in the presence of residual modes also. Steady state analyses will be performed to determine the influence of matrix  $E$  on the magnitude of vibration level and the control force. The objective of this analysis will be to determine an optimal  $E$  for given characteristics of magnetic bearings.

The applications of model reference, sliding mode adaptive control technique will also be investigated. In this approach, the sensor output can be made to behave like a response of a reference model having desired damping ratio and natural frequency. The rotor response at those locations where sensors are not mounted will be investigated.

Rotor parameters of space engines will be used in designing these control laws. Various implementation requirements such as power, current etc. will be estimated.

#### 3.2 EXPERIMENTAL VALIDATION OF THE CONTROL SYSTEMS

Two phases of hardware development will be carried out as follows:

Phase I - Test Stand Set Up and System Characterization

The rotor fixture, the magnetic bearings, the sensors and the controller will be integrated and a shakedown test will be performed. A series of tests will first be carried out to determine the magnetic bearings' dynamic characteristics. The rotor will be excited at various positions with impulsive, step, and periodic inputs, in order to characterize the structural dynamics. The results of these efforts will be used to validate the analytical model and modify the control law.

#### Phase II - Control Implementation and Validation

In the second phase of the experimental study, sensors and actuators for control purposes will be applied to the test stand according to the recommendations from the analytical work. The rigid rotor control algorithm will first be tested. System parameters, such as sensor locations, shaft length, shaft speeds, and rotor unbalanced force, will be varied systematically to examine the performance and robustness of the controller. The results from this phase will provide a quantitative measure of the efficacy of the proposed control strategy.

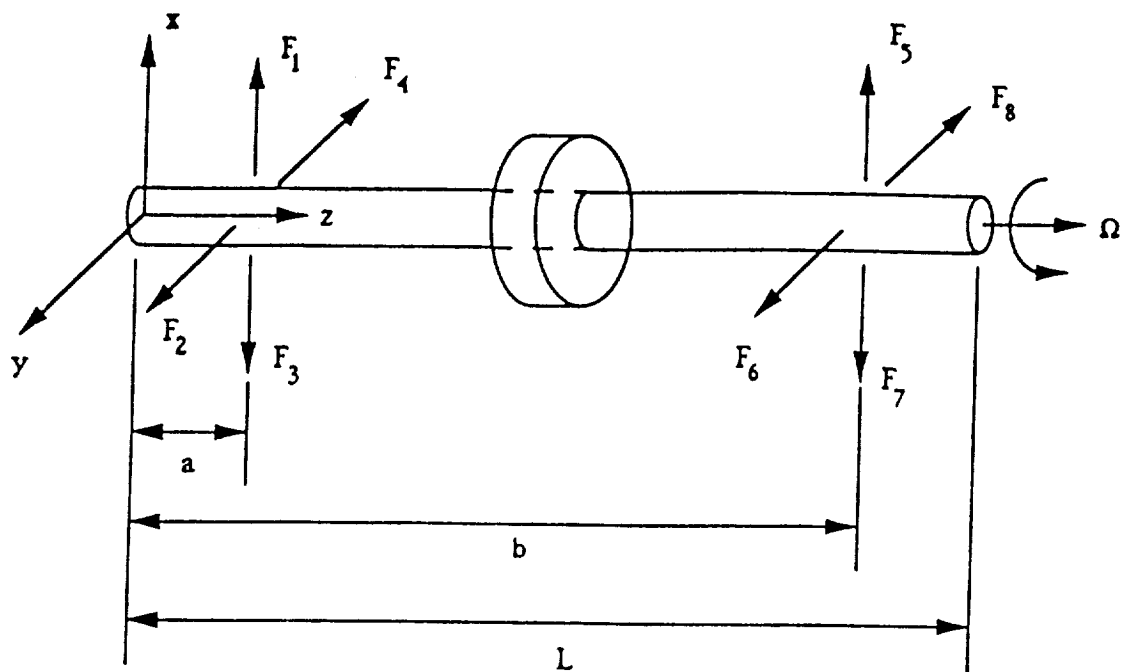


Figure 1: Generic Model of the Rotor Shaft

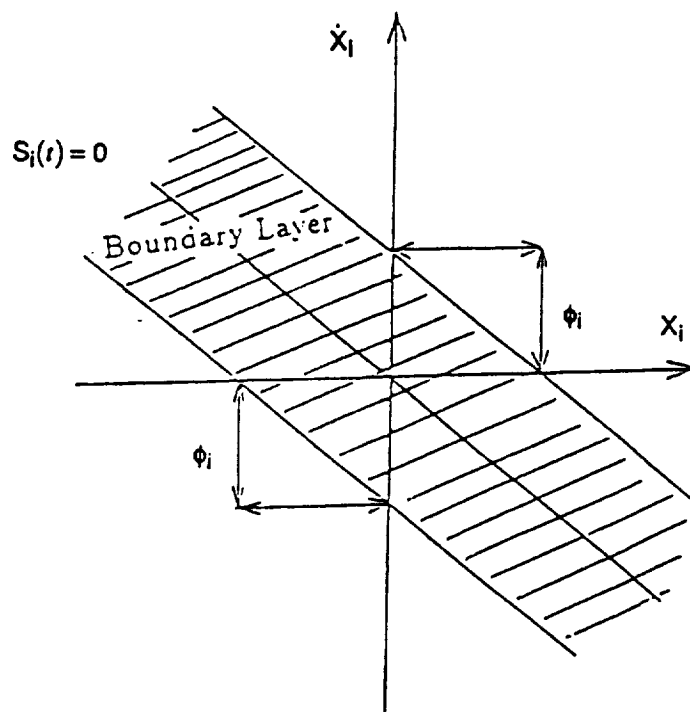


Figure 2: Illustration of Boundary Layer Concept



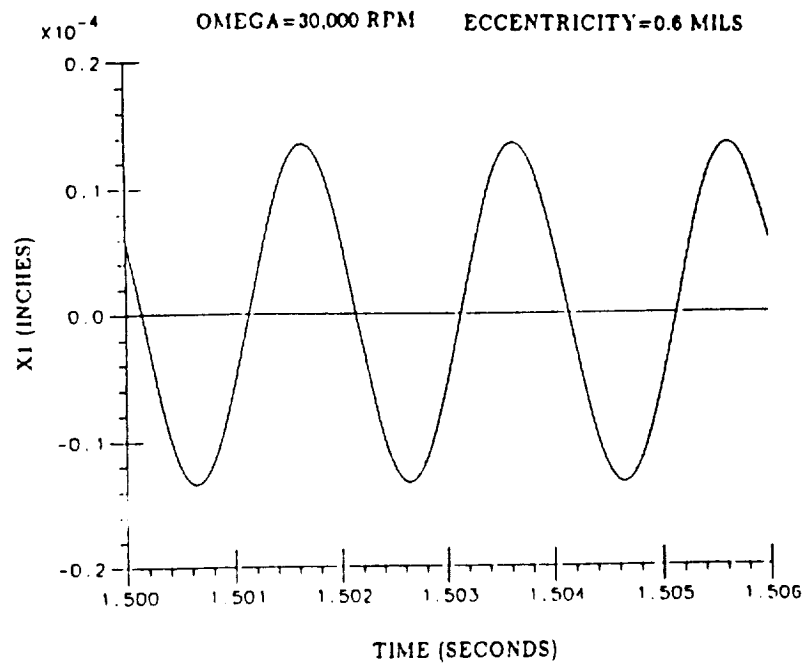


Figure 3: Steady State Response of the Controlled Rigid Rotor

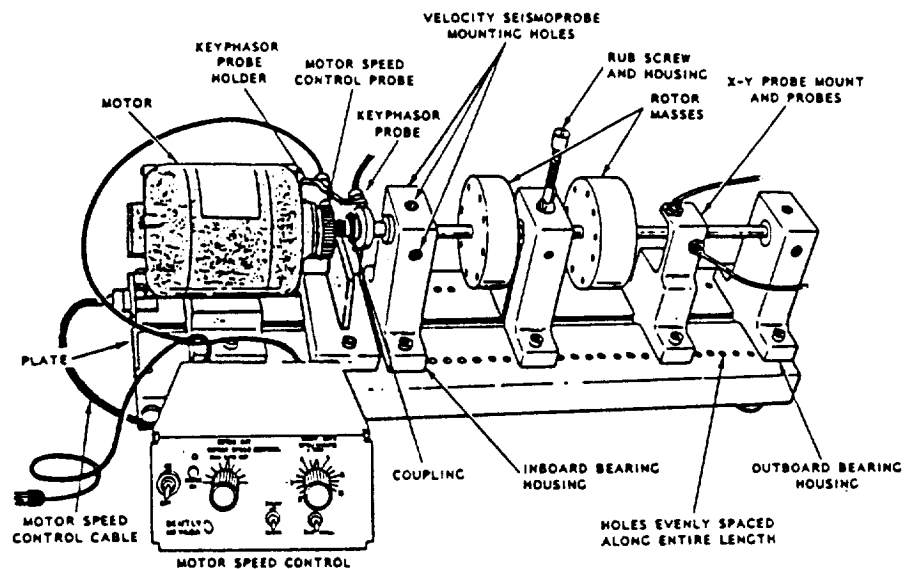


Figure 4: The Bentley-Nevada Rotor Fixture

